ORIGINAL INSHIHUB MATHEMATICS WORKBOOK

Independent and unofficial study resource

GATE Mathematics 2027 Revised-Syllabus Diagnostic

Original problems, error diagnosis, and a focused remediation route

Version 1.0 — 8 August 2026

Twenty original diagnostic problems with complete reasoning and targeted remediation.

InshiHub

What this workbook measures

This diagnostic isolates concepts that are newly named, broadened, or made more explicit in the official GATE 2027 Mathematics syllabus relative to the official 2026 list. Some broader parent topics, including Laplace transforms and local extrema, already appeared in 2026. This wording comparison does not claim first-time examinability; it identifies where a 2027 candidate should not rely on an old checklist alone.

Every problem, choice and solution below is original. The diagnostic is deliberately narrower than the complete Mathematics syllabus: a strong score here is evidence of readiness for the revised-focus concepts, not a prediction of rank or qualification.

Recommended attempt. Allow 50 minutes. Do not use notes. Record both your response and confidence (high, medium or low). MCQ items have one correct choice, MSQ items may have more than one correct choice, and NAT items require a number. Do not award partial credit on MSQ items.

How diagnosis differs from answer checking

The detailed review classifies each miss by cause: definition, theorem condition, counterexample, computation, or transfer. Fix the cause before repeating the item. A correct low-confidence answer still belongs in the review queue.

Source and correction discipline

The official 2027 MA syllabus and 2026 MA syllabus were compared and verified on 8 August 2026. Always check the current official site for later notices. Material workbook corrections are published at InshiHub Corrections.

Contents

Part I

Diagnostic paper

Attempt sheet

Set a 50-minute timer. Work through Questions 1–20 without notes, and record a confidence level for every response: H (high), M (medium), or L (low). For an MSQ item, select every correct option and no incorrect option; there is no partial credit in this workbook. Do not open the detailed solutions until the timer ends.

Scoring. Questions marked 1 carry one point; questions marked 2 carry two points. The maximum raw score is 32. A correct low-confidence response still enters the review queue because the aim is dependable recall, not a lucky total.

1 — A multivariable Taylor coefficient

Format: NAT, 1 mark Let $f(x, y) = e^{x+2y}$. In the Taylor polynomial of total degree 2 about $(0, 0)$, what is the coefficient of xy? **Response:** ________________ **Confidence:** [ ] H [ ] M [ ] L

2 — Classification at a degenerate critical point

Format: MCQ, 2 marks For

$$f(x, y) = (x^2 - y^2)^2,$$

which statement correctly classifies the origin?

A. It is a strict local maximum.

B. It is a strict local minimum.

C. It is a non-strict local minimum.

D. It is not a local extremum because the Hessian test is inconclusive.

Response: ________________ **Confidence:** [ ] H [ ] M [ ] L

3 — Change of variables over a rotated region

Format: NAT, 2 marks Let

$$D = \{(x, y) : |x + y| \le 1,\ |x - y| \le 1\}, \qquad I = \iint_D (x^2 + y^2)\, dA.$$

Find $3I$. **Response:** ____________________ **Confidence:** [ ] H [ ] M [ ] L

4 — An exponential factor in a Laplace transform

Format: NAT, 1 mark For $s > -2$, let

$$F(s) = \mathcal{L}\{te^{-2t}\}(s).$$

Find $(s + 2)^2 F(s)$. **Response:** ____________________ **Confidence:** [ ] H [ ] M [ ] L

5 — Consequences for a complete metric space

Format: MSQ, 2 marks Let X be a nonempty complete metric space and suppose $X = \bigcup_{n=1}^{\infty} F_n$, where every F_n is closed. Which statements must be true?

A. At least one F_n has nonempty interior.

B. Every F_n is dense in X.

C. X cannot be a countable union of nowhere-dense subsets.

D. X must be uncountable.

Response: ____________________ **Confidence:** [ ] H [ ] M [ ] L

6 — Testing uniform continuity

Format: MCQ, 1 mark Which function is continuous on the stated domain but not uniformly continuous there?

A. $f(x) = \sqrt{x}$ on $[0, \infty)$

B. $f(x) = x^2$ on $\mathbb{R}$

C. $f(x) = 1/x$ on $[1, \infty)$

D. $f(x) = \sin x$ on $\mathbb{R}$

Response: ________________ **Confidence:** [ ] H [ ] M [ ] L

7 — Membership at a singular endpoint

Format: MSQ, 2 marks On $(0, 1)$, let $f(x) = x^{-1/3}$. For which listed spaces does f belong?

A. $L^1(0, 1)$

B. $L^2(0, 1)$

C. $L^3(0, 1)$

D. $L^4(0, 1)$

Response: ________________ **Confidence:** [ ] H [ ] M [ ] L

8 — Computing an L2 norm

Format: NAT, 1 mark On $[0, 3]$, let $f(x) = x$. Find $\|f\|_2$. **Response:** ________________
Confidence: [ ] H [ ] M [ ] L

9 — Finite Abelian group classification

Format: NAT, 2 marks How many isomorphism classes of finite Abelian groups have order 72? **Response:** ____________________ **Confidence:** [ ] H [ ] M [ ] L

10 — Exact element orders

Format: NAT, 1 mark How many elements of order exactly 2 are in $\mathbb{Z}_4 \times \mathbb{Z}_2$? **Response:** ____________________ **Confidence:** [ ] H [ ] M [ ] L

11 — Operator classes on a Hilbert space

Format: MSQ, 2 marks Let H be an infinite-dimensional Hilbert space. Which statements are true?

A. The identity operator on H is bounded but not compact.

B. Every bounded finite-rank operator on H is compact.

C. Every compact linear operator from H to a normed space is bounded.

D. Every bounded linear operator on H is compact.

Response: ____________________ **Confidence:** [ ] H [ ] M [ ] L

12 — Separability of sequence spaces

Format: MCQ, 1 mark Which statement is false?

A. ℓ^2 is separable.

B. ℓ^∞ is separable.

C. Every finite-dimensional normed space is separable.

D. Every subspace of a separable normed space is separable.

Response: ________________ **Confidence:** [ ] H [ ] M [ ] L

13 — Norm of a functional

Format: NAT, 2 marks Equip $C([0,1])$ with the sup norm. Define

$$\phi(f) = f(1/4) - f(3/4).$$

Find the operator norm $\|\phi\|$. **Response:** ________________**Confidence:** [ ] H [ ] M [ ] L

14 — Classifying a diagonal operator

Format: MCQ, 2 marks Define $T : \ell^2 \to \ell^2$ by

$$T(x_1, x_2, x_3, ...) = (x_1, x_2/2, x_3/3, ...).$$

Which classification is correct?

A. T is compact and $\|T\| = 1$.

B. T is bounded with norm 1, but it is not compact.

C. T is unbounded.

D. T is compact and $\|T\| = 1/2$.

Response: ________________ **Confidence:** [ ] H [ ] M [ ] L

15 — A harmonic extremum from boundary data

Format: NAT, 1 mark Let u be harmonic on the open unit disk and continuous on its closure. Suppose

$$u(\cos\theta, \sin\theta) = 2 + \cos\theta$$

on the boundary. Find the maximum of u on the closed disk. **Response:** ____________________
Confidence: [ ] H [ ] M [ ] L

16 — Comparing two harmonic functions

Format: MCQ, 1 mark Let Ω be a bounded connected domain. Suppose u and v are harmonic on Ω, continuous on $\overline{\Omega}$, and agree on $\partial\Omega$. What must follow?

A. $u = v$ throughout Ω.

B. $u - v$ is a positive constant.

C. u and v may differ at interior critical points.

D. The conclusion holds only if the common boundary value is positive.

Response: ____________________ **Confidence:** [ ] H [ ] M [ ] L

17 — Path connectedness under constructions

Format: MSQ, 2 marks Which statements are true?

A. Every path-connected space is connected.

B. The continuous image of a path-connected space is path connected.

C. The closure of every path-connected subspace is path connected.

D. A countable product of path-connected spaces, with the product topology, is path connected.

Response: ____________________ **Confidence:** [ ] H [ ] M [ ] L

18 — Compactness criteria in metric spaces

Format: MSQ, 2 marks For a metric space X, which properties are equivalent to compactness?

- **A.** Every sequence in X has a convergent subsequence with limit in X.
- **B.** Every infinite subset of X has a limit point in X.
- **C.** X is complete.
- **D.** X is totally bounded.

Response: ____________________ **Confidence:** [ ] H [ ] M [ ] L

19 — A countable product topology

Format: MSQ, 2 marks Let $X = [0,1]^{\mathbb{N}}$ have the product topology. Which statements are true?

- **A.** X is compact.
- **B.** Every coordinate projection $\pi_n : X \to [0,1]$ is continuous.
- **C.** A basic open set restricts only finitely many coordinates.
- **D.** A basic open set must restrict every coordinate to a proper open subset of $[0,1]$.

Response: ____________________ **Confidence:** [ ] H [ ] M [ ] L

20 — Continuous extension from a closed set

Format: MCQ, 2 marks Let $X = [0,1]^2$, let $A = \partial X$, and let $g : A \to [-1,1]$ be continuous. Which conclusion is guaranteed?

- **A.** At least one continuous $G : X \to [-1,1]$ satisfies $G|_A = g$.
- **B.** Exactly one such continuous extension exists.
- **C.** A polynomial extension exists for every g.
- **D.** An extension exists only when g is constant on each edge.

Response: ____________________ **Confidence:** [ ] H [ ] M [ ] L

Response record

Copy each response and confidence level here before reading the solutions.

Q	Mark	Response / confidence	Q	Mark	Response / confidence
1	1		11	2	
2	2		12	1	
3	2		13	2	
4	1		14	2	
5	2		15	1	
6	1		16	1	
7	2		17	2	
8	1		18	2	
9	2		19	2	
10	1		20	2	

Stop here until the timer ends. The next part repeats each original item and gives a full derivation, a second-method check, a likely failure mode, and a targeted remediation route.

Part II

Detailed solutions

Diagnostic 1 — Mixed terms in a Taylor polynomial

Format: NAT, 1 mark **Focus:** Taylor's theorem

Original problem

Let $f(x,y) = e^{x+2y}$. In the Taylor polynomial of total degree 2 about $(0,0)$, what is the coefficient of xy?

Detailed solution

Write $z = x + 2y$. The second-degree expansion of e^z is

$$e^z = 1 + z + \frac{z^2}{2} + O(\|(x,y)\|^3).$$

Since

$$\frac{(x+2y)^2}{2} = \frac{x^2}{2} + 2xy + 2y^2,$$

the coefficient of xy is 2.

Final answer 2

Second-method check

The mixed part of the multivariable Taylor formula is

$$\frac{1}{2}\big(2f_{xy}(0,0)xy\big) = f_{xy}(0,0)xy.$$

Here $f_{xy}(0,0) = 2$, giving the same coefficient.

If you missed it — mixed-part factorial error
The factor $1/2$ multiplies both off-diagonal Hessian entries. Those two equal contributions cancel the apparent extra $1/2$; do not halve $f_{xy}xy$ again.

Remediation route

Rebuild the degree-two formula from $\frac{1}{2}h^T Hf(a)h$, expand one 2×2 Hessian explicitly, and then check it against a one-variable substitution.

Diagnostic 2 — A degenerate Hessian

Format: MCQ, 2 marks **Focus:** Taylor's theorem and local extrema

Original problem

For
$$f(x, y) = (x^2 - y^2)^2,$$
which statement correctly classifies the origin?

A. It is a strict local maximum.

B. It is a strict local minimum.

C. It is a non-strict local minimum.

D. It is not a local extremum because the Hessian test is inconclusive.

Detailed solution

For every (x, y),
$$f(x, y) = (x^2 - y^2)^2 \geq 0 = f(0, 0).$$
Thus the origin is a local minimum, indeed a global minimum. It is not strict: every nonzero point on either line $y = x$ or $y = -x$ also has value 0. Hence the correct classification is a non-strict local minimum.

Final answer C

Second-method check

The first nonzero homogeneous term in the expansion is the quartic $(x^2 - y^2)^2$. It is nonnegative but not positive definite. That is exactly the signature of a non-strict minimum in this example.

If you missed it — treating an inconclusive test as a negative result
A singular Hessian says only that the quadratic test does not decide the problem. It does not say that no extremum exists. Inspect the first nonzero term or the function itself.

Remediation route

Practise separating three conclusions: positive-definite Hessian, indefinite Hessian, and degenerate Hessian. For the third case, test signs and zero directions of the first nonzero Taylor term.

Diagnostic 3 — A Jacobian on a rotated square

Format: NAT, 2 marks **Focus:** Jacobians and change of variables

Original problem

Let

$$D = \{(x, y) : |x + y| \le 1,\ |x - y| \le 1\}$$

and

$$I = \iint_D (x^2 + y^2)\, dA.$$

Find $3I$.

Detailed solution

Set

$$u = x + y, \qquad v = x - y.$$

Then D becomes the square $[-1, 1]^2$, and

$$x = \frac{u+v}{2}, \qquad y = \frac{u-v}{2}, \qquad \left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{2}.$$

Also

$$x^2 + y^2 = \frac{u^2 + v^2}{2}.$$

Therefore

$$I = \int_{-1}^{1}\int_{-1}^{1} \frac{u^2+v^2}{4}\, du\, dv = \frac{1}{4}\left(\frac{4}{3} + \frac{4}{3}\right) = \frac{2}{3}.$$

Thus $3I = 2$.

Final answer 2

Second-method check

The region is a square with diagonals of length 2, hence area 2. Its symmetry gives equal x^2 and y^2 contributions. Direct slicing over the four congruent triangles also yields $I = 2/3$.

If you missed it — using the forward Jacobian
After expressing (x, y) as functions of (u, v), the area element needs $|\partial(x, y)/\partial(u, v)|$, not its reciprocal. The absolute value is mandatory.

Remediation route

For every substitution, write four lines before integrating: inverse map, new region, transformed integrand, and absolute inverse Jacobian.

Diagnostic 4 — A shifted Laplace transform

Format: NAT, 1 mark **Focus:** Laplace-transform properties

Original problem

For $s > -2$, let

$$F(s) = \mathcal{L}\{te^{-2t}\}(s).$$

Find $(s+2)^2 F(s)$.

Detailed solution

Since $\mathcal{L}\{t\}(s) = 1/s^2$, the exponential shift rule gives

$$\mathcal{L}\{e^{-2t}t\}(s) = \frac{1}{(s+2)^2}.$$

Hence the requested product is 1.

Final answer 1

Second-method check

From the definition,

$$F(s) = \int_0^\infty te^{-(s+2)t}\,dt = \frac{1}{(s+2)^2},$$

by one integration by parts.

If you missed it — sign error in the shift
Multiplication by e^{-at} replaces s with $s+a$. It does not replace s with $s-a$.

Remediation route

Derive the shift rule once from the defining integral, then check the sign by asking whether the extra exponential makes convergence easier or harder.

Diagnostic 5 — What Baire category actually guarantees

Format: MSQ, 2 marks **Focus:** Baire category theorem

Original problem

Let X be a nonempty complete metric space and suppose $X = \bigcup_{n=1}^{\infty} F_n$, where every F_n is closed. Which statements must be true?

A. At least one F_n has nonempty interior.

B. Every F_n is dense in X.

C. X cannot be a countable union of nowhere-dense subsets.

D. X must be uncountable.

Detailed solution

If every closed F_n had empty interior, then each would be nowhere dense. Their union would be a countable union of nowhere-dense sets equal to the nonempty complete metric space X, contradicting the Baire category theorem. Thus A is necessary, and C is the equivalent category statement.

B is not necessary: take $F_1 = X$ and $F_2 = \emptyset$. D is not necessary: a one-point metric space is nonempty, complete and finite.

Final answer A and C

Second-method check

Apply the dense-open form to $U_n = X \setminus F_n$. If every F_n were nowhere dense, every U_n would be open dense, so $\bigcap_n U_n$ would be dense and therefore nonempty. But its points would lie outside every F_n, contradicting the union assumption.

If you missed it — confusing category with cardinality
Baire category is about interior and nowhere-dense structure, not directly about how many points a space contains. Finite complete spaces are useful counterexamples to unsupported cardinality claims.

Remediation route

Memorise both equivalent forms of the theorem and build one finite counterexample whenever a proposed conclusion mentions countability of points rather than category.

Diagnostic 6 — Continuity is not uniform continuity

Format: MCQ, 1 mark **Focus:** Uniform continuity

Original problem

Which function is continuous on the stated domain but not uniformly continuous there?

- **A.** $f(x) = \sqrt{x}$ on $[0, \infty)$
- **B.** $f(x) = x^2$ on $\mathbb{R}$
- **C.** $f(x) = 1/x$ on $[1, \infty)$
- **D.** $f(x) = \sin x$ on $\mathbb{R}$

Detailed solution

For $x_n = n$ and $y_n = n + 1/n$, the input distance tends to zero, but

$$|y_n^2 - x_n^2| = 2 + \frac{1}{n^2} \not\to 0.$$

Therefore x^2 is not uniformly continuous on $\mathbb{R}$.

The other functions are uniformly continuous: $\sqrt{x}$ satisfies $|\sqrt{x} - \sqrt{y}| \leq \sqrt{|x-y|}$; $1/x$ is Lipschitz on $[1, \infty)$; and $\sin x$ is Lipschitz on $\mathbb{R}$.

Final answer B

Second-method check

The derivative of x^2 is unbounded, and the displayed sequences turn that growth into a direct violation. The bounded derivatives of $1/x$ and $\sin x$ give an immediate Lipschitz check.

If you missed it — using an unbounded domain as the criterion
Some functions are uniformly continuous on unbounded domains. The definition requires one δ to work everywhere; test whether local slopes can force a fixed output change over shrinking input gaps.

Remediation route

Practise three tools: Heine–Cantor on compact sets, bounded derivative implies Lipschitz, and the two-sequence contradiction used here.

Diagnostic 7 — Endpoint singularities in L^p

Format: MSQ, 2 marks **Focus:** L^p spaces

Original problem

On $(0,1)$, let $f(x) = x^{-1/3}$. For which listed spaces does f belong?

A. $L^1(0,1)$

B. $L^2(0,1)$

C. $L^3(0,1)$

D. $L^4(0,1)$

Detailed solution

For $1 \le p < \infty$,

$$\int_0^1 |f(x)|^p \, dx = \int_0^1 x^{-p/3} \, dx.$$

The integral converges exactly when $-p/3 > -1$, equivalently $p < 3$. Thus it converges for $p = 1, 2$ and diverges for $p = 3, 4$.

Final answer A and B

Second-method check

At $p = 3$ the integral is $\int_0^1 dx/x$, the logarithmic boundary case. Higher powers make the singularity stronger, so no $p \ge 3$ can work.

If you missed it — reversing finite-measure inclusions
On a finite-measure space, $L^q \subseteq L^p$ when $q > p$, but the endpoint test still decides the actual membership. Do not infer that belonging to L^1 implies belonging to every higher L^p.

Remediation route

Reduce power singularities to the template $\int_0^1 x^a \, dx < \infty \iff a > -1$, then mark the equality case separately.

Diagnostic 8 — An L^2-norm is not a squared norm

Format: NAT, 1 mark **Focus:** L^p spaces

Original problem

On $[0, 3]$, let $f(x) = x$. Find $\|f\|_2$.

Detailed solution

By definition,

$$\|f\|_2 = \left(\int_0^3 |x|^2\, dx\right)^{1/2} = \left(\frac{3^3}{3}\right)^{1/2} = 3.$$

Final answer 3

Second-method check

The squared norm is the inner product $\langle f, f\rangle = 9$; taking its positive square root gives 3.

If you missed it — reporting the energy instead of the norm
The integral of $|f|^2$ is $\|f\|_2^2$, not the norm itself.

Remediation route

Write the outer exponent $1/p$ before computing any L^p-norm.

Diagnostic 9 — Counting finite Abelian group types

Format: NAT, 2 marks **Focus:** Finite Abelian groups

Original problem

How many isomorphism classes of finite Abelian groups have order 72?

Detailed solution

Factor

$$72 = 2^3 \cdot 3^2.$$

For a fixed prime p, Abelian groups of order p^k correspond to partitions of k. If $P(k)$ denotes the number of partitions of k, there are $P(3) = 3$ possibilities for the 2-primary part and $P(2) = 2$ for the 3-primary part. Primary components combine independently, so the total is

$$P(3)P(2) = 3 \cdot 2 = 6.$$

Final answer 6

Second-method check

The 2-parts are $\mathbb{Z}_8$, $\mathbb{Z}_4 \times \mathbb{Z}_2$, and $\mathbb{Z}_2^3$. The 3-parts are $\mathbb{Z}_9$ and $\mathbb{Z}_3^2$. Pairing them gives six distinct primary decompositions.

If you missed it — counting presentations instead of isomorphism classes
The classification is organised prime by prime and by partitions of exponents. Different-looking products can be isomorphic when coprime cyclic factors are combined.

Remediation route

Factor the order, list partitions of each prime exponent, and multiply the partition counts only after the primary lists are complete.

Diagnostic 10 — Elements of exact order two

Format: NAT, 1 mark **Focus:** Finite Abelian groups

Original problem

How many elements of order exactly 2 are in $\mathbb{Z}_4 \times \mathbb{Z}_2$?

Detailed solution

An element (a, b) has order dividing 2 exactly when

$$2a = 0 \pmod 4, \qquad 2b = 0 \pmod 2.$$

Thus $a \in \{0, 2\}$ and $b \in \{0, 1\}$, giving four elements killed by 2. Remove the identity $(0, 0)$. The remaining three elements all have order exactly 2.

Final answer 3

Second-method check

They are explicitly $(2, 0)$, $(0, 1)$, and $(2, 1)$. Doubling each gives the identity, and none is the identity.

If you missed it — forgetting exact order
Solving $2g = 0$ counts elements whose order divides 2, including the identity. Exact order requires removing the order-one case.

Remediation route

First count the kernel of multiplication by 2; then subtract all elements already counted at a proper divisor of the target order.

Diagnostic 11 — Bounded does not mean compact

Format: MSQ, 2 marks **Focus:** Bounded and compact operators

Original problem

Let H be an infinite-dimensional Hilbert space. Which statements are true?

A. The identity operator on H is bounded but not compact.

B. Every bounded finite-rank operator on H is compact.

C. Every compact linear operator from H to a normed space is bounded.

D. Every bounded linear operator on H is compact.

Detailed solution

The identity is bounded with norm 1. If (e_n) is an orthonormal sequence, then its image under the identity has no convergent subsequence, so the identity is not compact. Thus A is true and D is false.

A bounded finite-rank operator sends bounded sets into bounded subsets of a finite-dimensional range. Their closures are compact, so B is true.

A compact linear operator is bounded: otherwise one can choose $\|x_n\| \le 1$ with $\|Tx_n\| \to \infty$, whose image cannot have a convergent subsequence. Thus C is true.

Final answer A, B and C

Second-method check

On a finite-dimensional normed space, bounded and relatively compact sets coincide after closure. In infinite dimensions, Riesz's lemma produces bounded sequences separated by a fixed distance; the identity preserves that separation.

If you missed it — collapsing two operator classes
Compactness controls the image of every bounded sequence through subsequences. Boundedness controls only its size. The identity is the standard separation example.

Remediation route

For each operator claim, test an orthonormal sequence and a finite-rank truncation before invoking a theorem.

Diagnostic 12 — Which familiar space is not separable?

Format: MCQ, 1 mark **Focus:** Separability

Original problem

Which statement is false?

A. ℓ^2 is separable.

B. ℓ^∞ is separable.

C. Every finite-dimensional normed space is separable.

D. Every subspace of a separable normed space is separable.

Detailed solution

Finite sequences with rational real and imaginary parts form a countable dense subset of ℓ^2. Finite-dimensional normed spaces are separable, and subspaces of separable metric spaces are separable.

By contrast, for every subset $A \subseteq \mathbb{N}$, its indicator sequence $\mathbf{1}_A$ lies in ℓ^∞. Distinct indicators have sup-distance 1. This is an uncountable 1-separated set, which cannot be contained in a separable metric space. Thus ℓ^∞ is not separable.

Final answer B

Second-method check

If a countable set were dense in ℓ^∞, open balls of radius $1/3$ about it would cover the space. Each such ball can contain at most one indicator sequence, contradicting the uncountable family above.

If you missed it — transferring the ℓ^2 picture to ℓ^∞
Finite truncations approximate a sequence in ℓ^2, but they need not converge in the sup norm. The metric, not just the underlying sequences, determines separability.

Remediation route

Build a countable dense set when you expect separability; build an uncountable uniformly separated set when you expect nonseparability.

Diagnostic 13 — The norm of an evaluation difference

Format: NAT, 2 marks **Focus:** Dual spaces

Original problem

Equip $C([0,1])$ with the sup norm. Define

$$\phi(f) = f(1/4) - f(3/4).$$

Find the operator norm $\|\phi\|$.

Detailed solution

For every f,

$$|\phi(f)| \le |f(1/4)| + |f(3/4)| \le 2\|f\|_\infty.$$

Hence $\|\phi\| \le 2$.

Choose a continuous piecewise-linear function with $f(1/4) = 1$, $f(3/4) = -1$, and $|f(x)| \le 1$ everywhere. Then $\|f\|_\infty = 1$ and $|\phi(f)| = 2$. Thus the upper bound is attained.

Final answer 2

Second-method check

In the measure representation of $C([0,1])^*$, the functional corresponds to the signed measure $\delta_{1/4} - \delta_{3/4}$, whose total variation is $1 + 1 = 2$.

If you missed it — stopping after an upper bound
An operator-norm estimate is not an exact value until a unit vector or an approximating sequence reaches the bound.

Remediation route

Use the two-step norm template: prove a universal upper bound, then design a unit-norm input that aligns all terms with the desired signs.

Diagnostic 14 — A compact infinite-rank diagonal operator

Format: MCQ, 2 marks **Focus:** Compact operators

Original problem

Define $T : \ell^2 \to \ell^2$ by

$$T(x_1, x_2, x_3, \dots) = (x_1, x_2/2, x_3/3, \dots).$$

Which classification is correct?

- **A.** T is compact and $\|T\| = 1$.
- **B.** T is bounded with norm 1, but it is not compact.
- **C.** T is unbounded.
- **D.** T is compact and $\|T\| = 1/2$.

Detailed solution

The operator norm is

$$\|T\| = \sup_{n\geq 1} \frac{1}{n} = 1,$$

with equality at the first unit vector.

Let T_N retain only the first N coordinates of T. Each T_N has finite rank, and

$$\|T - T_N\| = \sup_{n>N} \frac{1}{n} = \frac{1}{N+1} \longrightarrow 0.$$

Thus T is a norm limit of compact finite-rank operators and is compact.

Final answer A

Second-method check

For the unit ball, all image tails satisfy

$$\sum_{n>N} \frac{|x_n|^2}{n^2} \leq \frac{1}{(N+1)^2}.$$

Uniformly small tails plus finite-dimensional compactness of the first N coordinates gives a direct subsequence proof.

If you missed it — equating infinite rank with noncompactness
Compact operators may have infinite rank. For diagonal maps on ℓ^2, diagonal coefficients tending to zero are the key compactness signal.

Remediation route

Approximate diagonal operators by finite-rank truncations and compute the tail operator norm.

Diagnostic 15 — Boundary data control a harmonic maximum

Format: NAT, 1 mark **Focus:** Maximum Principle

Original problem

Let u be harmonic on the open unit disk and continuous on its closure. Suppose

$$u(\cos\theta, \sin\theta) = 2 + \cos\theta$$

on the boundary. Find the maximum of u on the closed disk.

Detailed solution

By the Maximum Principle, the maximum on the closed disk occurs on the boundary. The boundary function $2 + \cos\theta$ has maximum 3. Hence the closed-disk maximum is 3.

Final answer 3

Second-method check

The harmonic function $v(x, y) = 2 + x$ has exactly the stated boundary values. Uniqueness for the Dirichlet problem gives $u = v$, whose maximum on the closed disk is visibly 3 at $(1, 0)$.

If you missed it — searching for interior critical points first
A nonconstant harmonic function cannot attain its maximum in the interior. Use the boundary before doing calculus.

Remediation route

Apply the principle to both u and $-u$, then reduce the problem to an elementary boundary optimisation.

Diagnostic 16 — Uniqueness from the Maximum Principle

Format: MCQ, 1 mark **Focus:** Maximum Principle

Original problem

Let Ω be a bounded connected domain. Suppose u and v are harmonic on Ω, continuous on $\overline{\Omega}$, and agree on $\partial\Omega$. What must follow?

- **A.** $u = v$ throughout Ω.
- **B.** $u - v$ is a positive constant.
- **C.** u and v may differ at interior critical points.
- **D.** The conclusion holds only if the common boundary value is positive.

Detailed solution

The difference $w = u - v$ is harmonic and has boundary value 0. The Maximum Principle gives $w \leq 0$ in Ω. Applying it to $-w$ gives $-w \leq 0$, hence $w \geq 0$. Therefore $w = 0$ everywhere.

Final answer A

Second-method check

If w were nonzero, either its maximum would be positive or its minimum would be negative. Both extrema are excluded by the corresponding boundary value 0.

If you missed it — applying the theorem only to nonnegative functions
The Maximum Principle has no positivity hypothesis on the harmonic function. Apply it to both signs when proving uniqueness.

Remediation route

Turn equal-boundary problems into zero-boundary problems by subtracting the two candidate solutions.

Diagnostic 17 — Path connectedness and its permanence

Format: MSQ, 2 marks **Focus:** Path connectedness

Original problem

Which statements are true?

A. Every path-connected space is connected.

B. The continuous image of a path-connected space is path connected.

C. The closure of every path-connected subspace is path connected.

D. A countable product of path-connected spaces, with the product topology, is path connected.

Detailed solution

A path between any two points rules out a separation, so A is true. If γ is a path and f is continuous, then $f \circ \gamma$ is a path, so B is true.

C is false. The graph

$$\{(x, \sin(1/x)) : 0 < x \leq 1\}$$

is path connected, but its closure adds the vertical segment at $x = 0$ and is the topologist's sine curve, which is not path connected.

For D, choose coordinatewise paths γ_n between the corresponding coordinates. The map $t \mapsto (\gamma_n(t))_n$ is continuous in the product topology because every coordinate map is continuous. Thus D is true.

Final answer A, B and D

Second-method check

Connectedness and path connectedness are both preserved by continuous images, while the explicit closure example separates their behaviour. The universal property of the product topology verifies the product path.

If you missed it — assuming closure preserves paths
Closure preserves connectedness, but new limit points need not be reachable by a continuous path.

Remediation route

Keep a theorem table for continuous images, products, closures and components; attach one counterexample to every failed permanence rule.

Diagnostic 18 — Three compactness notions in metric spaces

Format: MSQ, 2 marks **Focus:** Sequential and limit-point compactness

Original problem

For a metric space X, which properties are equivalent to compactness?

A. Every sequence in X has a convergent subsequence with limit in X.

B. Every infinite subset of X has a limit point in X.

C. X is complete.

D. X is totally bounded.

Detailed solution

In metric spaces, compactness, sequential compactness, and limit-point compactness are equivalent. Thus A and B are correct.

Completeness alone is not enough: $\mathbb{R}$ is complete and not compact. Total boundedness alone is not enough: $(0, 1)$ is totally bounded but not complete and not compact. The correct combined criterion is completeness plus total boundedness.

Final answer A and B

Second-method check

From a sequence with no convergent subsequence one can extract an infinite set without a limit point. Conversely, a limit point of the range of a sequence yields a convergent subsequence after removing repeated trivial cases. These metric arguments connect A and B.

If you missed it — using only half of the metric compactness criterion
Complete and totally bounded work together. Each by itself has a standard counterexample.

Remediation route

Memorise the metric-space equivalence triangle and pair $\mathbb{R}$ with $(0, 1)$ as the two one-sided counterexamples.

Diagnostic 19 — What Tychonoff says about a countable cube

Format: MSQ, 2 marks **Focus:** Tychonoff theorem

Original problem

Let $X = [0,1]^{\mathbb{N}}$ have the product topology. Which statements are true?

- **A.** X is compact.
- **B.** Every coordinate projection $\pi_n : X \to [0,1]$ is continuous.
- **C.** A basic open set restricts only finitely many coordinates.
- **D.** A basic open set must restrict every coordinate to a proper open subset of $[0,1]$.

Detailed solution

Each factor $[0,1]$ is compact, so Tychonoff's theorem makes their product compact. Coordinate projections are continuous by the definition of the product topology.

A standard basic open set has the form

$$\prod_{n=1}^{\infty} U_n,$$

where each U_n is open in $[0,1]$ and $U_n = [0,1]$ for all but finitely many n. Thus C is true and D describes the finer box topology, not the product topology.

Final answer A, B and C

Second-method check

The product topology is the coarsest topology making all projections continuous. Its finite-coordinate subbasis both proves B and explains the finite restriction in C; compactness then follows directly from Tychonoff.

If you missed it — confusing product and box bases
Allowing every coordinate to be restricted creates the box topology. Tychonoff's compactness theorem concerns the product topology.

Remediation route

Write a basic neighbourhood of a point in each topology and circle the coordinates allowed to differ from the whole factor.

Diagnostic 20 — Extending boundary data continuously

Format: MCQ, 2 marks **Focus:** Tietze extension theorem

Original problem

Let $X = [0,1]^2$, let $A = \partial X$, and let $g : A \to [-1,1]$ be continuous. Which conclusion is guaranteed?

- **A.** At least one continuous $G : X \to [-1,1]$ satisfies $G|_A = g$.
- **B.** Exactly one such continuous extension exists.
- **C.** A polynomial extension exists for every g.
- **D.** An extension exists only when g is constant on each edge.

Detailed solution

The square X is a metric space and therefore normal. Its boundary A is closed. The bounded Tietze extension theorem therefore provides a continuous extension $G : X \to [-1,1]$ of g.

The theorem gives existence, not uniqueness or polynomial regularity, and it places no edgewise-constancy condition on g.

Final answer A

Second-method check

For nonuniqueness, take $g = 0$. Both $G_1 = 0$ and $G_2(x,y) = x(1-x)y(1-y)$ vanish on the boundary and are continuous on X, but they differ inside. This also shows why uniqueness is not part of Tietze's statement.

If you missed it — adding conclusions that the theorem never states
Tietze provides a bounded continuous extension from a closed subspace of a normal space. It does not promise uniqueness, smoothness, or a formula.

Remediation route

Check the three operative hypotheses—normal ambient space, closed subspace, continuous real-valued data—and state only the existence and range-control conclusion.

Part III

Scoring, diagnosis and remediation

Answer key and raw score

Award the listed mark only for a fully correct response. For an MSQ item, that means selecting every correct option and no incorrect option. The raw score measures only the revised-focus concepts sampled here; it is not a rank, qualifying-score, or full-syllabus prediction.

Q	Mark	Key	Diagnostic module
1	1	2	Taylor expansion
2	2	C	Taylor expansion and extrema
3	2	2	Change of variables
4	1	1	Laplace transforms
5	2	A, C	Baire category
6	1	B	Uniform continuity
7	2	A, B	L^p membership
8	1	3	L^2-norm
9	2	6	Finite Abelian groups
10	1	3	Finite Abelian groups
11	2	A, B, C	Bounded and compact operators
12	1	B	Separability
13	2	2	Dual norm
14	2	A	Compact diagonal operators
15	1	3	Maximum Principle
16	1	A	Harmonic uniqueness
17	2	A, B, D	Path connectedness
18	2	A, B	Metric compactness
19	2	A, B, C	Product topology and Tychonoff
20	2	A	Tietze extension
Maximum		**32**	

Interpret the result

Score	Band	What the evidence supports
27–32	Delta-ready	The sampled revised-focus ideas are mostly retrievable. Review every miss and every low-confidence correct response.
21–26	Nearly ready	Several ideas are usable, but at least one weak module can still cause avoidable losses.
14–20	Partial coverage	Recognition exceeds dependable execution. Rebuild the weakest modules before taking a mixed retest.
7–13	Major gaps	The revised-focus layer is not yet stable. Use short concept blocks followed by immediate exit tests.
0–6	Foundation rebuild	Start from definitions and canonical examples; delay mixed timed work until single-module accuracy improves.

The band is deliberately conservative. Move down one band for planning purposes if three or more correct responses had low confidence, because the same total may not reproduce under exam pressure.

Module diagnosis

For each row, add the marks you earned on the listed questions. Treat 80% or more as stable, 50%–79% as fragile, and below 50% as a rebuild priority. A one-question module is only a signal and must be confirmed with additional practice.

Module	Questions	Maximum	First repair action
Taylor and Jacobians	1–3	5	Expand each quadratic form and verify the inverse-map Jacobian.
Laplace transforms	4	1	Re-derive the exponential shift from the defining integral.
Analysis foundations	5–8	6	Pair every theorem with its hypotheses and one endpoint or sequence counterexample.
Finite Abelian groups	9–10	3	Factor prime-by-prime, list exponent partitions, then test exact order via kernels.
Functional analysis	11–14	7	Separate boundedness, compactness and separability with standard sequences and finite-rank truncations.
Maximum Principle	15–16	2	Reduce extrema and uniqueness to boundary statements, applying the principle to both signs.
Topology	17–20	8	Build a permanence table for images, products and closures; distinguish product from box topology.

Classify the cause, not only the topic

Label every miss and every low-confidence correct response with one primary cause:

Definition A symbol or property could not be stated precisely.

Condition The right theorem was recalled but a hypothesis, direction, or boundary case was omitted.

Counterexample A false generalisation survived because no standard separating example was available.

Computation The method was sound but an algebraic, Jacobian, norm, or counting step failed.

Transfer The idea was familiar in isolation but not recognised in the form used by the question.

Repair the cause before repeating the item. Re-reading a solution rarely fixes a missing counterexample or an unchecked theorem condition.

Fourteen-day remediation route

1. **Day 0:** complete this timed attempt and tag every review item by module, confidence, and cause.
2. **Days 1–4:** rebuild the two lowest-percentage modules. End each session with two closed-book examples, not a reread.
3. **Days 5–7:** solve new single-module items under short time limits. Require both a correct answer and a written theorem-condition or verification line.

4. **Day 8:** redo only the missed and low-confidence items here, in a shuffled order, without consulting the previous work.
5. **Days 9–12:** mix the repaired modules with older syllabus material so recognition does not depend on a topic label.
6. **Day 14:** take a fresh 50-minute mixed diagnostic. A module graduates only after at least 80% accuracy with no low-confidence correct response.

Evidence boundary. These 20 original questions sample concepts newly named, broadened, or made more explicit in the official 2027 Mathematics syllabus relative to the 2026 list. Some broader parent topics were already listed in 2026. The questions do not reproduce official material, guarantee topic weight, or establish full GATE readiness.